S13C - 0433

A new approach to measure time- and frequency-dependent seismic travel-time and velocity evolution

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1. Background

- Goal: measure the evolution of seismic travel-time shift (or velocity change) in order to monitor the temporal changes in the elastic properties of crustal medium
- Use repeating earthquakes or ambient noise cross-correlations, direct wave or coda
- From homogeneous change to localized change:

i. Horizontal resolution: station pair average

ii. Depth resolution: different frequency bands

iii. 3D spatial resolution: both lapse-time and frequency resolution needed

- Current methods

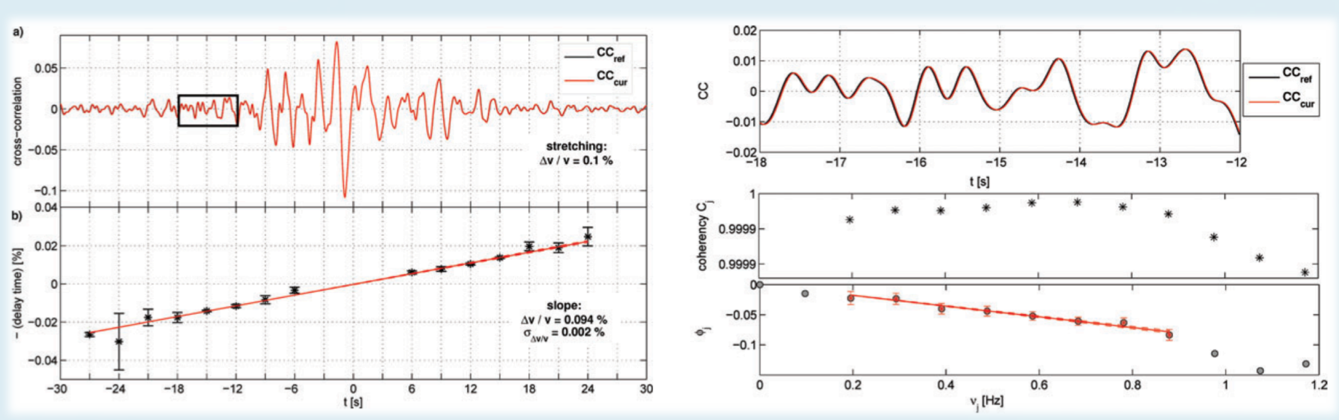
i. Time domain: Windowed CC, Stretching, and Dynamic Time Warping, but

-----> affected by changes in amplitude spectra

ii. Frequency domain: cross-spectrum by moving-window Fourier transform, or, continuous wavelet transform?

2. Methods to estimate travel-time shifts

2.1 Traditional approach: Moving-Window Cross-Spectrum (MWCS) Method



Assumptions:
- Homogeneous change
- Constant dt within each moving window

Advantages:
- High precision
- Not biased by changes in amplitude spectra of sources
- Can easily resolve clock error

To be improved:
- Lapse-time resolution
- Frequency resolution
- Computational cost
- Cycle skipping

Figure 1. Principles of MWCS technique proposed by Poupinet *et al*., (1984), figure from Clarke *et al*., (2011)

2.2 New approach: Wavelet Cross-Spectrum (WCS) Method

- **3 Steps to estimate dt (dv/v) by wavelet cross-spectrum**

Continuous Wavelet Transform (CWT): $W_x(a,b) = \frac{1}{|a|^{1/2}}\int_{-\infty}^{\infty} x(t)\ \psi^*(\frac{t-b}{a})dt$

Continuous Wavelet Cross-Spectrum (CWCS): $W_{xy}(a,b) = W_x^*(a,b)W_y(a,b)$

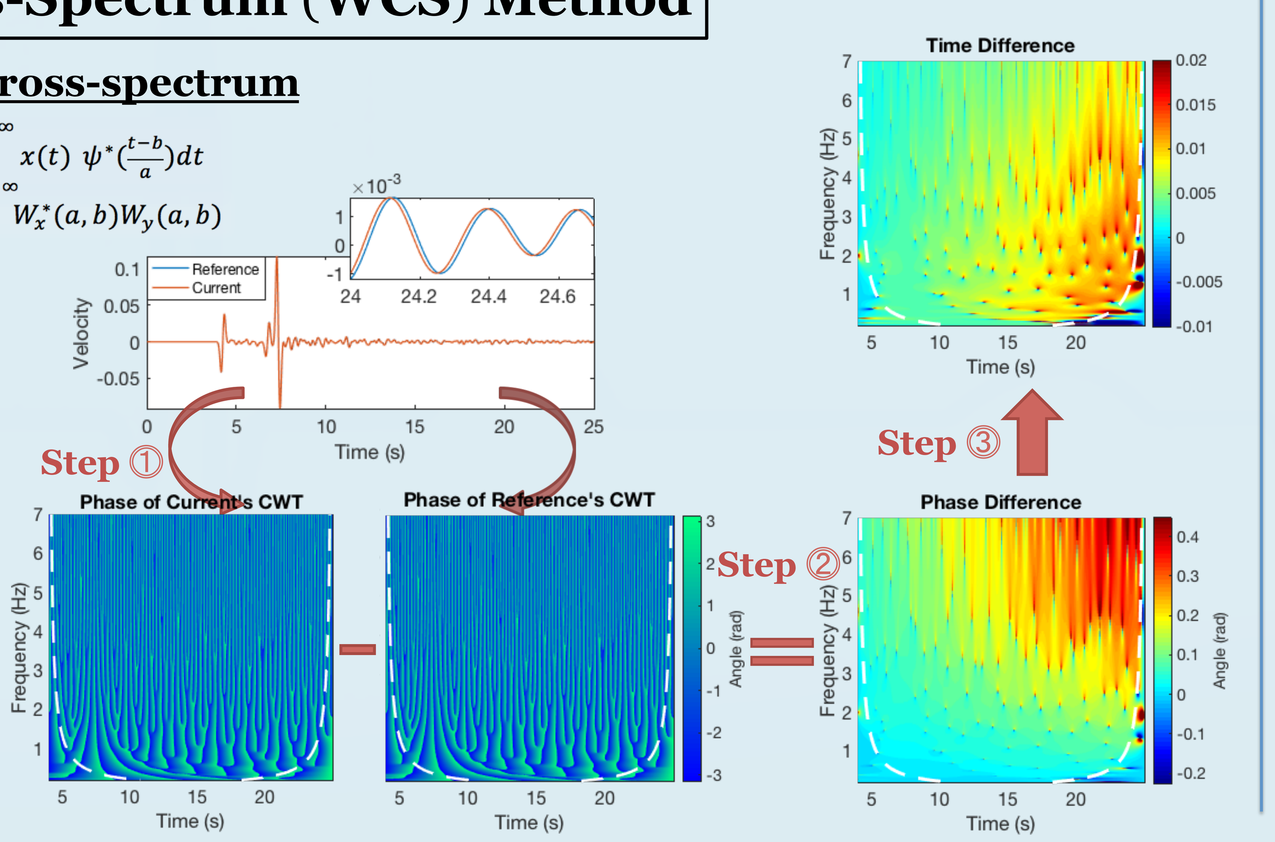
→ **Step ①** : Perform CWT for both the reference and current waveforms.

→ **Step ②** : Calculate the CWCS and take its phase (*i.e.* the phase difference between the two CWTs).

→ **Step ③** : Divide the phase difference by 2π×frequency to obtain the time difference at each point over the whole time-frequency domain.

Now play with the dt!

e.g. homogeneous change: linear regression of dt over t at each frequency using coda waves,
or, time delay at each frequency using direct waves

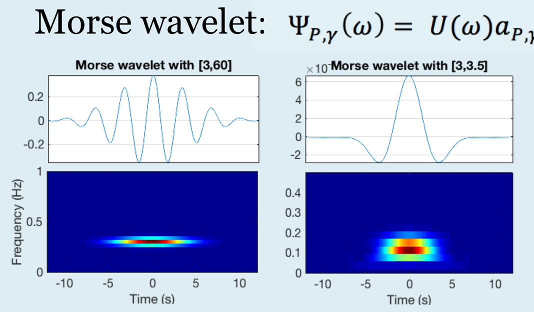


- **Choice of wavelet function**

Frequency domain definition

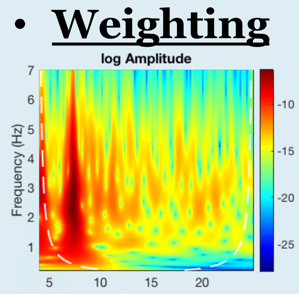
Morlet wavelet: $\Psi_\sigma(\omega) = C_\sigma \pi^{-\frac{1}{4}}\left(e^{-\frac{1}{2}(\sigma-\omega)^2} - \kappa_\sigma e^{-\frac{1}{2}\omega^2}\right)$

Morse wavelet: $\Psi_{P,\gamma}(\omega) = U(\omega)a_{P,\gamma}\omega^{\frac{P^2}{\gamma}}e^{-\omega\gamma}$



- Analytic wavelet
- Different spreads in time and frequency axis, but: $\sigma_t\sigma_\omega \geq \frac{1}{2}$
- Trade-off between time and frequency resolution

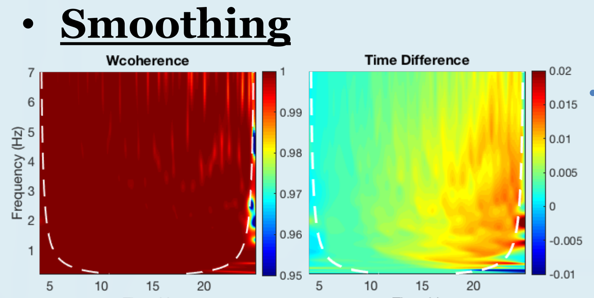
- **Weighting**



- The phase is not well-defined when the amplitude of the waveform small.

→ Weight the time difference by amplitude of the WCS (*i.e.* the amplitude product of two CWTs).

- **Smoothing**



- By proper smoothing, the coherence of the wavelet cross-spectrum can be used to select high-quality data.

- **Advantages of the WCS Method:**

(besides those of the MWCS Method …)

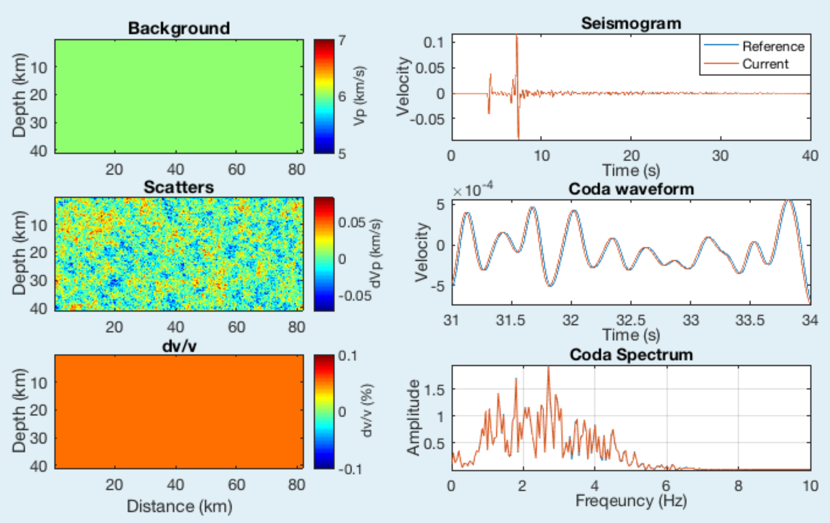
- Computationally efficient
- Gives time-shift all over the time-frequency domain
- Applies automatically adaptive window length
- Allows the use of frequency dependent lapse-time window

3. Synthetic tests

3.1 Synthetic coda

- **Homogeneous velocity change**

- Simulations of synthetic coda by a 2D Finite Difference Method (Li *et al*., 2014)
- Free surface at topside, absorbing boundaries for the other three sides
- Reference model: homogeneous background velocity structure, plus scatters by von-Karman auto-correlation function
- Current model: after a 0.05% homogeneous velocity change from the reference model over the whole medium



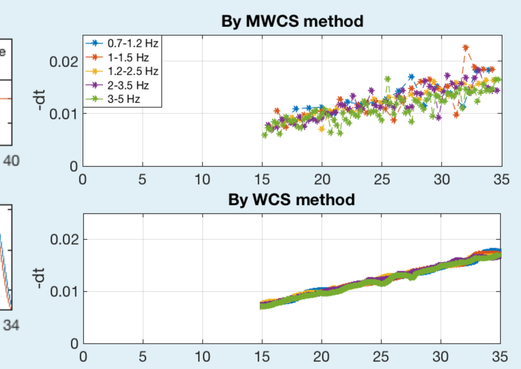


Figure 2. Comparison of travel-time delay vs. lapse time measurements by the two methods (average for five frequency bands respectively).

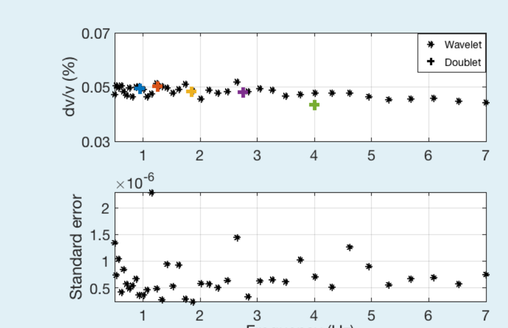
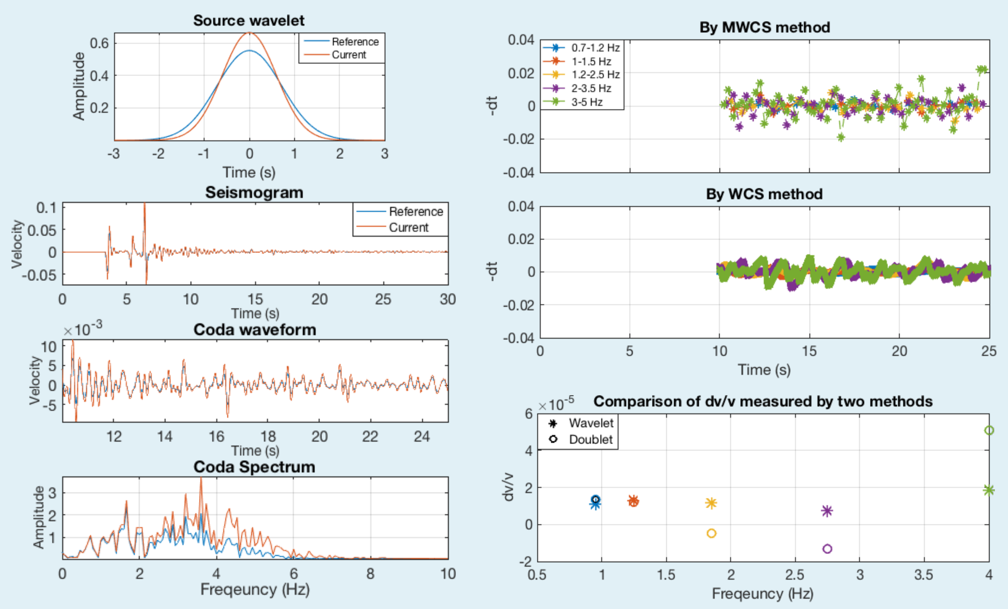


Figure 3. Comparison of dv/v measured by the two methods. MWCS method gives average dv/v within a frequency range, while WCS method gives dv/v at each frequency all at once.

- **Change in Source Spectra**



Reference and current seismograms: generated with the same velocity model, but Gaussian sources of different bandwidth

→No velocity change theoretically

3.2 Synthetic dispersive waves

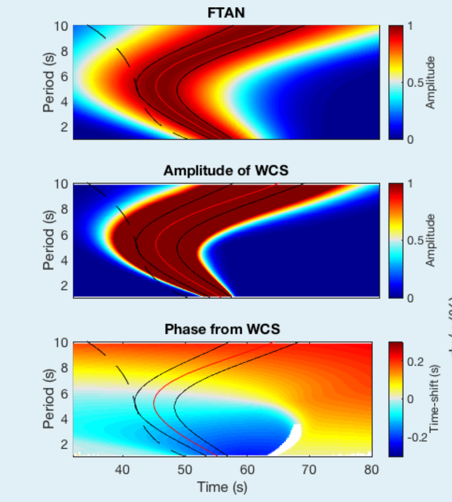
- Synthetic surface waves generated by two slightly different phase velocity dispersion curves:

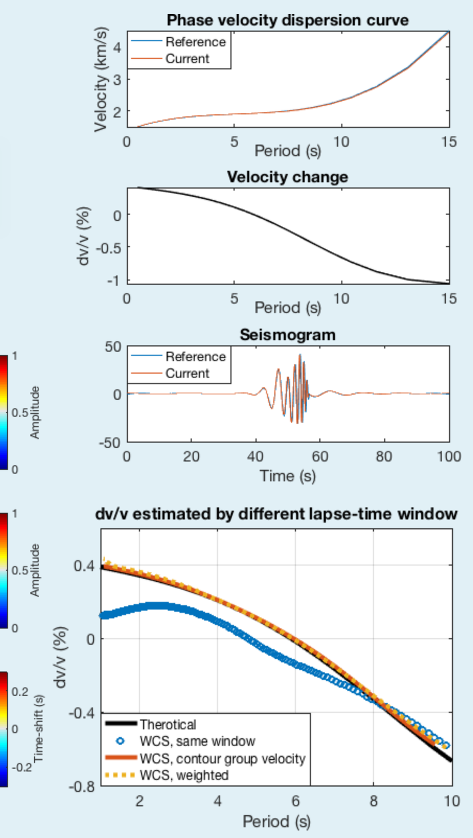
$C_0(T) = -5.1*10^{-5}*T^4 + 6.383*10^{-3}*T^3 - 0.09*T^2 + 0.5*T + 2.788$

$C_1(T) = -5.1001*10^{-5}*T^4 + 6.380*10^{-3}*T^3 - 0.0905*T^2 + 0.501*T + 2.80$

- Velocity change not homogeneous, but varies with depth (*i.e.* period)

- Energy in CWTs and CWCS concentrates around the group velocity dispersion curve
- Wavelet cross-spectrum method allows the use of evolving lapse-time window at different period





4. Real data application

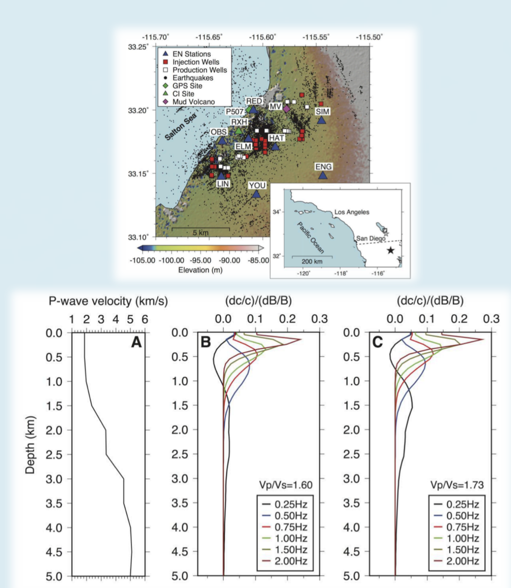
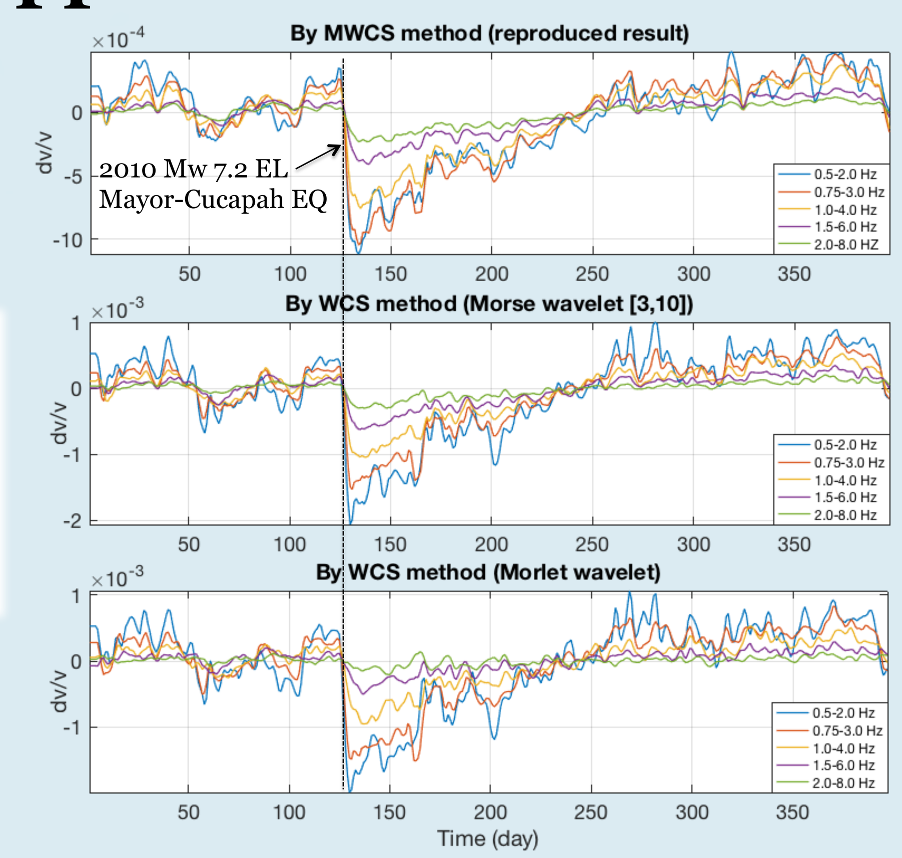
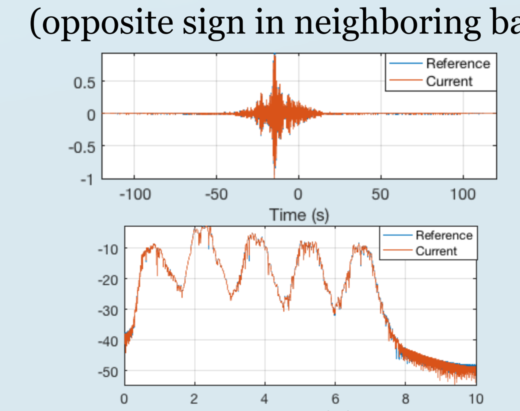


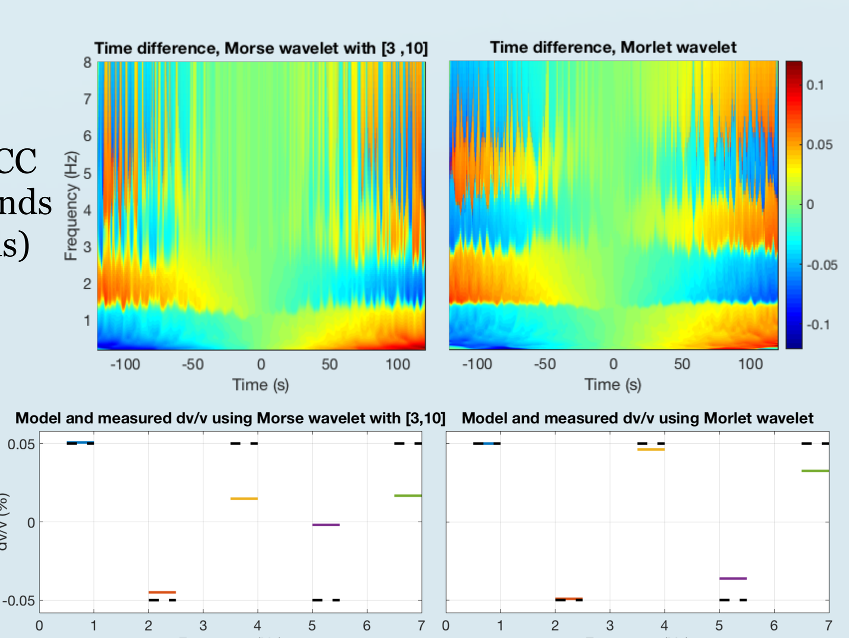
Figure 4. Seismicity map and Rayleigh-wave phase velocity sensitivity at Salton Sea Geothermal Field from Taira *et al*., (2018)



1D Checker-Board Test

- Reference: a realistic CC at SSGF
- Current: stretch/compress the ref CC by 0.05% in different frequency bands (opposite sign in neighboring bands)





5. Conclusions

- We propose a new approach (WCS) to estimate the time- and frequency-dependent travel time shifts and velocity changes based on wavelet-cross-spectrum analysis.
- This new method is computationally efficient and gives dt measurements all over time and frequency domain with best resolution under uncertainty principle.
- Synthetic tests show that the WCS method can more stably and accurately retrieve the travel time shifts than the traditional method, and is able to deal with dispersive wave scenarios.
- An application on real data at the Salton Sea Geothermal Field shows that the WCS method better separates the dv/v at low frequencies, and is less affected by smearing at high frequencies.

References

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Acknowledgements

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